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Research Article

Cardy-Verlinde Formula and Its Self-Gravitational Corrections for Regular Black Holes

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We check the consistency of the entropy of Bardeen and Ayón Beato-García-Bronnikov black holes with the entropy of particular conformal field theory via Cardy-Verlinde formula. We also compute the first-order semiclassical corrections of this formula due to self-gravitational effects by modifying pure extensive and Casimir energy in the context of Keski-Vakkuri, Kraus and Wilczek analysis. It is concluded that the correction term remains positive for both black holes, which leads to the violation of the holographic bound.

1. Introduction

The idea of massive bodies from which nothing can escape due to strong gravity was firstly proposed in 1783 by Michell [1] and named as "dark stars." In 1967, Wheeler [2] used the term "black hole" for such objects. Black hole (BH) is the most important prediction of general relativity which is detected by its effect on the nearby matter. A BH can be defined as a spacetime singularity which is covered by an event horizon. Hawking [3] found that BHs also radiate particles from their boundary with finite thermal spectrum dubbed after his name as Hawking radiations. Several efforts have been made to visualize this spectrum by examining the quantum effects on a scalar particle [4]. Taking quantum effects into account, BH evaporation converts pure quantum states into mixed states known as "information loss paradox" [5]. Much work has been done to resolve this paradox using fixed geometrical background during emission process.

In 1995, Kraus and Wilczek [6, 7] put forward an analysis in which dynamical BH background was used by treating the Hawking radiations as a tunneling process. They found that Hawking radiations are modified and their spectrum becomes nonthermal after considering self-gravitational interaction. In this procedure, total mass (Arnowitt-Deser-Misner (ADM)) remains constant, while the mass of the Schwarzschild BH decreases due to emitted radiations. A specific Painlevé coordinate transformation [8] is used in order to avoid the existence of singularity at the horizon. Initially, this analysis was implemented on spherically symmetric geometry (Schwarzschild type BHs $A(r) = B^{-1}(r)$) to evaluate their corrected temperature and entropy. An extra nonthermal term appears in thermodynamical quantities due to this modification. The modified BH temperature also depends upon the emitted shell energy (ω) along with BH characteristics, while entropy is different from the Bekenstein-Hawking entropy (S) for the corresponding Schwarzschild BH.

Vagenas [9] applied Keski-Vakkuri, Kraus and Wilczek (KKW) analysis on non-Schwarzschild type BH using more generalized coordinate transformation. This transformation allows studying the across horizon BH physics whose generalization leads to exact expressions of thermodynamical quantities. The temperature and entropy of non-Schwarzschild type BH are not any more the Hawking temperature (T_H) and Bekenstein-Hawking entropy, respectively. Parikh and Wilczek [10] were the first to utilize this technique to four-dimensional Schwarzschild BH. Later, this analysis was used for several BHs such as $(d + 1)$-dimensional anti-de Sitter (AdS) BH, 2-dimensional AdS BH, two-dimensional charged and uncharged dilatonic BH, $(2 + 1)$-dimensional charged BTZ BH, spinning BTZ BH, and magnetic stringy BH [11–17].

Holographic principle proposed by Susskind [18] is one of the fundamental principles of quantum gravity. It states that Bekenstein-Hawking entropy should be larger than the entropy associated with volume $V(S \leq \mathscr{A}/4)$, where $G = c = \hbar = 1$. Generally, radiations can be described by using interacting conformal field theory (CFT) and the AdS/CFT correspondence is the most common example of the holography. Verlinde [19] proposed a universal formula (Cardy-Verlinde formula (CV)) which relates the entropy of a certain CFT (S_{CFT}) to its total energy (E) and Casimir energy (E_C) valid for all dimensions. The generalization of this formula is the major outcome of AdS/CFT correspondence. This formula remains valid for topological dS, Schwarzschild-dS, Reissner-Nordström-dS, Kerr-dS, Kerr-Newman-dS BHs, and so forth [20–26].

The issue of quantum corrections in entropy attracted many researchers [27–29]. Carlip [30] deduced the leading order quantum correction to the classical CV formula. Setare and Vagenas [31, 32] proved that this formula holds for Achúcarro-Ortiz BH and calculated the self-gravitational corrections in S_{CFT} in the framework of KKW analysis. Setare and Jamil [33] showed that the entropy of the charged rotating BTZ BH can be expressed in terms of S_{CFT}. Darabi et al. [34] found the self-gravitational corrections in this formula for the charged BTZ BH. Recently, Abbas [35] showed that the entropy of noncommutative Schwarzschild BH can be expressed in terms of CV formula.

In this paper, we study entropy of the Bardeen and Ayón Beato-García-Bronnikov (ABGB) BHs in terms of CV formula and also find the first-order leading self-gravitational corrections. The paper is organized as follows. In Section 2, we calculate quantities like mass, potential, temperature, and entropy of both BHs. We prove that the entropies of these BHs coincide with that of CFT via CV formula. Section 3 is devoted to the computation of the self-gravitational corrections in the CFT entropy of these BHs using KKW analysis. Finally, we summarize the results in the last section.

2. Entropy of Regular Black Holes via Cardy-Verlinde Formula

In this section, we evaluate the entropy of the Bardeen and ABGB BHs through the Cardy-Verlinde formula.

2.1. Entropy of Bardeen Black Hole. The four-dimensional spherically symmetric BH is represented by the following line element:

$$ds^2 = -Adt^2 + A^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{1}$$

where

$$A(r) = 1 - 2\frac{M(r)}{r}. \tag{2}$$

For different choices of $M(r)$, this reduces to some well-known BHs. Bardeen [36] proposed a model obeying weak energy condition that can be interpreted as the solution of magnetic monopole. The mass function has the following specific choice for Bardeen BH:

$$M(r) = \frac{mr^3}{\left(r^2 + Q^2\right)^{3/2}}, \tag{3}$$

with mass m and charge Q. For zero charges, this mass function leads to the Schwarzschild BH. The lapse function $A(r) = 0$ provides a unique root ($r_+ = 2M(r_+)$) corresponding to the event horizon (outer horizon). Substituting this value of $M(r)$, the event horizon takes the form

$$r_+ = 2M(r_+) = 2\frac{mr_+^3}{\left(r_+^2 + Q^2\right)^{3/2}}, \tag{4}$$

leading to the value of m as

$$m = \frac{\left(r_+^2 + Q^2\right)^{3/2}}{2r_+^2}. \tag{5}$$

The corresponding electric potential (Φ_+) and the Hawking temperature can be calculated as

$$\begin{aligned} \Phi_+ &= \left.\frac{\partial M}{\partial Q}\right|_{r=r_+} = -\frac{3}{2}\left(\frac{Qr_+}{r_+^2 + Q^2}\right), \\ T_H &= \left.\frac{1}{4\pi}\frac{dA}{dr}\right|_{r=r_+} = \frac{1}{4\pi r_+}\left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right). \end{aligned} \tag{6}$$

The corresponding area and entropy are given as

$$\begin{aligned} \mathscr{A} &= \int \sqrt{g_{\theta\theta}g_{\phi\phi}}\, d\theta\, d\phi = 4\pi r_+^2, \\ S &= \frac{\mathscr{A}}{4} = \pi r_+^2 = \pi\left(2\frac{mr_+^3}{\left(r_+^2 + Q^2\right)^{3/2}}\right)^2. \end{aligned} \tag{7}$$

The generalized Cardy-Verlinde formula is defined as [19]

$$S_{\text{CFT}} = \frac{2\pi R}{\sqrt{ab}}\sqrt{E_C\left(2E - E_C\right)}, \tag{8}$$

where a and b are arbitrary positive constants and R is the radius of the sphere. The Casimir energy is generated by quantum fluctuations of CFT at finite volume while it disappears when volume becomes infinite. Casimir effects are usually significant at zero temperatures but can also be discussed at its finite values of the temperature. Thus, an extensive (in thermodynamical system, the energy $E(S,V)$ is called extensive when it satisfies the relation $E(\lambda S, \lambda V) = \lambda E(S,V)$) part is added in the total energy expressed as

$$E(S,V) = E_E(S,V) + \frac{1}{2}E_C(S,V), \tag{9}$$

where E_E is the pure extensive part of energy and the factor $1/2$ is used for convenience [19].

The violation of Euler identity is given as

$$E = V\left(\frac{\partial E}{\partial V}\right)_S + S\left(\frac{\partial E}{\partial S}\right)_V, \tag{10}$$

which provides n-dimensional Casimir energy in the following form:

$$E_C = n\left(E + PV - TS - \Phi_+ Q\right), \tag{11}$$

where pressure is defined as $P = E/nV$. The product of energy and radius (ER) is independent of volume due to conformal invariance which is satisfied by both E_E and E_C. Hence, both energies can be expressed in terms of R and S in arbitrary dimensions as follows [19]:

$$E_C = \frac{b}{2\pi R} S^{1-1/n}, \qquad E_E = \frac{a}{4\pi R} S^{1+1/n}. \tag{12}$$

Taking $n = 1$ and using the above expressions of P, T_H, S, Φ_+ in (11), the Casimir energy takes the form

$$E_C = 2E - \frac{r_+}{4}\left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) + \left(\frac{3Q^2 r_+}{r_+^2 + Q^2}\right). \tag{13}$$

Using mass energy relation ($E = M$), this reduces to

$$E_C = 2M - \frac{r_+}{4}\left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) + \left(\frac{3Q^2 r_+}{r_+^2 + Q^2}\right). \tag{14}$$

The extensive energy is found through (9) and (13) as

$$2E_E = 2E - E_C = -\frac{r_+}{4}\left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) + \left(\frac{3Q^2 r_+}{r_+^2 + Q^2}\right). \tag{15}$$

Using (7), the generalized formulas of Casimir and extensive energy in one dimension can be written as

$$E_C = \frac{b}{2\pi R}, \qquad E_E = \frac{\pi a}{4R} r_+^4. \tag{16}$$

Comparing the Casimir as well as extensive energy given in (14)–(16), we obtain two different spherical radii as

$$\begin{aligned} R &= \pi a r_+^4 \left[\frac{\left(r_+^2 + Q^2\right)}{2\pi r_+ \left(r_+^2 - 2Q^2\right) - 3Q^2 r_+}\right], \\ R &= \frac{b}{\pi} \\ &\quad \times \left[\frac{\left(r_+^2 + Q^2\right)}{4M\left(r_+^2 + Q^2\right) - 2r_+\left(r_+^2 - 2Q^2\right) - 3Q^2 r_+}\right]. \end{aligned} \tag{17}$$

Multiplication of the above two equations provides

$$\begin{aligned} R &= \left(abr_+\left(r_+^2 + Q^2\right)^2 \right. \\ &\quad \times \left(\left(2\pi r_+\left(r_+^2 - 2Q^2\right) - 3Q^2 r_+\right)\right. \\ &\quad \left.\left. \times \left(4M\left(r_+^2 + Q^2\right) - 2r_+\left(r_+^2 - 2Q^2\right) - 3Q^2 r_+\right)\right)^{-1}\right)^{1/2}. \end{aligned} \tag{18}$$

Substituting (14), (15), and (18) in CV formula, the final result is

$$S_{\mathrm{CFT}} = \pi r_+^2 = S. \tag{19}$$

This shows that the entropy of Bardeen BH can be expressed in terms of CV formula in the context of CFT.

2.2. Entropy of Ayón Beato-García-Bronnikov Black Hole. Ayón Beato and García [37] and Bronnikov [38] proposed a nonsingular BH by solving the system of equations coupled with nonlinear electrodynamics and gravity. For this BH, the mass function is given as

$$M(r) = m\left[1 - \tanh\left(\frac{Q^2}{2mr}\right)\right], \tag{20}$$

which also reduces to the Schwarzschild BH for $Q = 0$. The event horizon has the following form:

$$r_+ = 2M\left(r_+\right) = 2m\left[1 - \tanh\left(\frac{Q^2}{2mr_+}\right)\right]. \tag{21}$$

The corresponding electric potential, Hawking temperature, and entropy turn out to be

$$\begin{aligned} \Phi_+ &= \left.\frac{\partial M}{\partial Q}\right|_{r=r_+} = -\frac{Q}{r_+}\sec h^2\left(\frac{Q^2}{2mr_+}\right), \\ T_H &= \left.\frac{1}{4\pi}\frac{dA}{dr}\right|_{r=r_+} = \frac{1}{4\pi r_+^2}\left(r_+ + Q\Phi_+\right), \\ S &= \frac{\mathcal{A}}{4} = \pi r_+^2 = \pi\left(2m\left[1 - \tanh\left(\frac{Q^2}{2mr_+}\right)\right]\right)^2. \end{aligned} \tag{22}$$

Using (11), the Casimir energy can be written as

$$E_C = 2E - \frac{1}{4}r_+ + \frac{5}{4}\left(\frac{Q^2}{r_+}\sec h^2\left(\frac{Q^2}{2mr_+}\right)\right). \tag{23}$$

For $E = M$, this reduces to

$$E_C = \frac{3}{2}M + \frac{5}{4}\left(\frac{Q^2}{r_+}\sec h^2\left(\frac{Q^2}{2mr_+}\right)\right). \tag{24}$$

The extensive energy is given as

$$2E_E = 2E - E_C = \frac{1}{4}r_+ - \frac{5}{4}\left(\frac{Q^2}{r_+}\sec h^2\left(\frac{Q^2}{2mr_+}\right)\right). \tag{25}$$

Comparison of (24) and (25) with (16) leads to

$$\begin{aligned} R &= \frac{\pi a}{4} r_+^4 \left[\frac{1}{(1/8)\,r_+ - (5/8)\left(\left(Q^2/r_+\right)\sec h^2\left(Q^2/2mr_+\right)\right)}\right], \\ R &= \frac{b}{2\pi}\left[\frac{1}{(3/2)\,M + (5/4)\left(\left(Q^2/r_+\right)\sec h^2\left(Q^2/2mr_+\right)\right)}\right], \end{aligned} \tag{26}$$

yielding the unique radius

$$\begin{aligned} R &= \frac{\sqrt{ab}}{2\sqrt{2}} r_+^2 \\ &\quad \times \left[1 \times \left(\left(\frac{3}{2}M + \frac{5}{4}\left(\frac{Q^2}{r_+}\sec h^2\left(\frac{Q^2}{2mr_+}\right)\right)\right)\right.\right. \\ &\quad \left.\left. \times \left(\frac{1}{8}r_+ - \frac{5}{8}\left(\frac{Q^2}{r_+}\sec h^2\left(\frac{Q^2}{2mr_+}\right)\right)\right)\right)^{-1/2}\right]. \end{aligned} \tag{27}$$

Inserting (24)–(27) in (8), we find that the entropy of CFT is equivalent to the entropy of ABGB BH as

$$S_{\mathrm{CFT}} = \pi r_+^2 = S. \tag{28}$$

3. Self-Gravitational Corrections to the Cardy-Verlinde Formula

In this section, we evaluate the self-gravitational corrections in the CV formula for both BHs using KKW analysis. This leads to modifying all the quantities in the above section under self-gravitational effects.

3.1. Self-Gravitational Corrections for Bardeen Black Hole. Here, we evaluate the self-gravitational corrections in CV formula for Bardeen BH. The modified Casimir energy ($\widetilde{E}_C$) is defined as

$$\widetilde{E}_C = 2E - T_{\mathrm{bh}} S_{\mathrm{bh}} - Q_{\mathrm{bh}} \Phi_{\mathrm{bh}}, \tag{29}$$

where T_{bh} and S_{bh} stand for modified temperature and entropy, respectively. The third term of (11) is modified to $T_{\mathrm{bh}} S_{\mathrm{bh}}$, whereas the total energy, charge, radius of the sphere, and electric potential of BH remain invariant under this effect. In order to evaluate the corrections in BH temperature and entropy, we use the KKW analysis. In this analysis, the total mass is kept fixed, while BH mass is assumed to fluctuate because a shell of energy ω radiates massless particles and hence BH mass reduces to $M - \omega$. This shell energy does not contain charge due to which electric potential remains the same; that is, $Q_{\mathrm{bh}} \Phi_{\mathrm{bh}} = Q\Phi_+$. Applying this analysis, we obtain a relationship between BH entropy and Bekenstein-Hawking entropy as

$$S_{\mathrm{bh}} = S - 4\pi M^2 \left[1 - \left(1 - \frac{\omega}{M}\right)^2\right]. \tag{30}$$

The corresponding self-gravitational corrected temperature is given by

$$T_{\mathrm{bh}} = \frac{\omega}{4\pi M^2} \left[1 - \left(1 - \frac{\omega}{M}\right)^2\right]^{-1}, \tag{31}$$

which reduces to the Hawking temperature by considering corrections up to first-order in ω.

Thus, the self-gravitational corrections in $T_{\mathrm{bh}} S_{\mathrm{bh}}$ take the form

$$T_{\mathrm{bh}} S_{\mathrm{bh}} = T_H S - \frac{2}{r_+} \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) M\omega. \tag{32}$$

Substituting this value in (29), the modified Casimir energy turns out to be

$$\begin{aligned} \widetilde{E}_C &= 2M - \frac{r_+}{4} \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) + \frac{3}{2} \left(\frac{r_+ Q^2}{r_+^2 + Q^2}\right) \\ &\quad - \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) M\omega \\ &= E_C + \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) M\omega, \end{aligned} \tag{33}$$

where the second term is the correction term. The modification in extensive part of the energy can be obtained by adding the term $2E$ in the above expression as

$$\begin{aligned} \widetilde{E}_E &= \frac{r_+}{8} \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) - \frac{3}{4} \left(\frac{r_+ Q^2}{r_+^2 + Q^2}\right) - \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) M\omega \\ &= E_E - \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) M\omega. \end{aligned} \tag{34}$$

The extensive energy (16) can also be modified in the context of KKW analysis as follows:

$$\widetilde{E}_E = \frac{a}{4\pi R} S_{\mathrm{bh}}^2 = \frac{\pi a}{4R} r_{\mathrm{out}}^4 = \frac{\pi a}{4R} r_+^4 \left(1 - \frac{\omega}{M}\right). \tag{35}$$

Inserting (18), (33), and (34) in modified CV formula, that is,

$$S_{\mathrm{CFT}} = \frac{2\pi R}{\sqrt{ab}} \sqrt{\widetilde{E}_C \left(2E - \widetilde{E}_C\right)}, \tag{36}$$

it follows that

$$\begin{aligned} S_{\mathrm{CFT}} = S \Bigg(& \Big(\Big(8Mr_+ \left(r_+^2 + Q^2\right) - r_+^2 \left(r_+^2 - 2Q^2\right) \\ & + \frac{64}{r_+} \left(r_+^2 + Q^2\right) M^2 \omega + 8r_+ \left(r_+^2 - 2Q^2\right) M\omega \\ & - 8 \left(r_+^2 - 2Q^2\right) M\omega \Big) + 12Q^2 r_+^2 + 36Q^4 r_+^2 \Big) \\ & \times \left[\frac{\left(r_+^2 - 2Q^2\right)^{-2}}{\left(4M\left(r_+^2 + Q^2\right) - \alpha\right)\alpha} \right] \Bigg)^{1/2}, \end{aligned} \tag{37}$$

where $\alpha = (2\pi r_+ (r_+^2 - 2Q^2) + 3Q^2 r_+)$. This gives the corrections in CV formula due to the effects of self-gravitation.

The total energy is the combination of extensive and Casimir energies, so $E > E_C$, which implies that the term $2E - E_C$ is positive. As the correction term is positive, so the modified term $\widetilde{E}_C(2E - \widetilde{E}_C)$ is greater in magnitude than the original one. The self-gravitational correction of CV formula for Bardeen BH remains positive as the term inside square root is real and positive. The above term can be expanded up to the first-order in ω as follows:

$$\begin{aligned} \widetilde{E}_C \left(2E - \widetilde{E}_C\right) &= E_C \left(2E - E_C\right) \\ &\quad - \left(E_C - \left(2E - E_C\right)\right) \left(\frac{r_+^2 - 2Q^2}{r_+^2 + Q^2}\right) M\omega, \end{aligned} \tag{38}$$

which relates the modified self-gravitational energy with the original energy. It is seen that the self-gravitational modified temperature and entropy derived in the context of KKW analysis of Bardeen BH are different from the Bekenstein-Hawking temperature and entropy, respectively. The BH entropy described by CV formula violates the holographic bound as $S_{\mathrm{CFT}} > S > S_{\mathrm{bh}}$ [39, 40].

3.2. Self-Gravitational Corrections for Ayón Beato-García-Bronnikov Black Hole. Now, we consider ABGB BH to calculate the self-gravitational corrections in the CV formula using KKW analysis. The corresponding self-gravitational corrections in $T_{\text{bh}}S_{\text{bh}}$ are

$$T_{\text{bh}}S_{\text{bh}} = T_H S - \frac{\omega}{2M}\left(r_+ + Q\Phi_+\right). \tag{39}$$

Equation (29) gives the modified Casimir energy as

$$\begin{aligned}\widetilde{E}_C &= \frac{3}{2}M + \frac{5}{4}\left(\frac{Q^2}{r_+}\sec h^2\left(\frac{Q^2}{2mr_+}\right)\right) + \frac{2}{r_+^2}\left(r_+ + Q\Phi_+\right)M\omega \\ &= E_C + \frac{2}{r_+^2}\left(r_+ + Q\Phi_+\right)M\omega,\end{aligned} \tag{40}$$

where the second term is the correction term. Additionally, the modified extensive energy can be written as

$$\begin{aligned}2E - \widetilde{E}_C &= 2E - E_C - \frac{2}{r_+^2}\left(r_+ + Q\Phi_+\right)M\omega, \\ \widetilde{E}_E &= E_E - \frac{1}{r_+^2}\left(r_+ + Q\Phi_+\right)M\omega.\end{aligned} \tag{41}$$

Inserting (27), (40), and (41) in modified CV formula, we obtain the self-gravitational corrected CV formula for ABGB BH as follows:

$$\begin{aligned}&S_{\text{CFT}} \\ &= S\left[\left(\left(\frac{3}{8}Mr_+ - \frac{4}{r_+^2}\left(r_+ + Q\Phi_+\right)M^2\omega\right.\right.\right. \\ &\qquad\left.\left. - \frac{25}{16}\frac{Q^4}{r_+^2}\sec h^4\left(\frac{Q^2}{2mr_+}\right)\right)\right) \\ &\quad\times\left(\left(\frac{1}{8}r_+ - \frac{5}{8}\frac{Q^4}{r_+^2}\sec h^4\left(\frac{Q^2}{2mr_+}\right)\right)\right) \\ &\qquad\left.\times\left(\frac{3}{2}M + \frac{5}{4}\frac{Q^4}{r_+^2}\sec h^4\left(\frac{Q^2}{2mr_+}\right)\right)\right)^{-1}\Bigg)^{1/2}\Bigg].\end{aligned} \tag{42}$$

As the correction term $(2/r_+^2)(r_+ + Q\Phi_+) > 0$, this implies that $\widetilde{E}_C(2E - \widetilde{E}_C) > E_C(2E - E_C)$. The corresponding first-order expansion of the correction term is

$$\begin{aligned}\widetilde{E}_C\left(2E - \widetilde{E}_C\right) &= E_C\left(2E - E_C\right) \\ &\quad + \frac{2}{r_+^2}\left(r_+ + Q\Phi_+\right)\left(2E - E_C\right)M\omega.\end{aligned} \tag{43}$$

This shows that the ABGB BH entropy described by CV formula also violates the holographic bound.

4. Concluding Remarks

The description of BH entropy by CV formula and its quantum corrections have attained much attention. There have been a number of papers to find the semiclassical corrections in this formula for different BHs using various techniques such as self-gravitational effects [41], generalized uncertainty principle [42], space noncommutativity [43], and quantum corrections [44]. The self-gravitational correction allows one to study across horizon physics and provides the solution of information loss paradox.

This paper is devoted to study this analysis for the Bardeen and ABGB BHs which correspond to the Schwarzschild BH for zero charges. Firstly, we have evaluated the thermodynamical quantities such as electric potential, Hawking temperature, and Bekenstein-Hawking entropy of both of the BHs. It is found that there exists unique event horizon whose entropy can be expressed in the form of CV formula indicating that this formula holds on both event horizons.

Secondly, we have used the self-gravitational effect to evaluate the temperature and entropy of these BHs in the context of KKW analysis. The corrections in the CV formula due to self-gravitational effects are evaluated by taking dynamical background of the BH. The corrections are restricted up to the linear order in ω; for the zeroth order, the modified temperature corresponds to the Hawking temperature. We have also found the corrections in all the quantities which can be modified under self-gravitational effect, that is, pure extensive energy and the Casimir energy. The electric potential, charge, spherical radius, and total energy remain invariant under this effect. It turns out that the self-gravitational correction term in modified CV formula is positive for both BHs as the modified term is greater than the original. It is interesting to mention here that the positive self-gravitational corrections for regular BHs do not satisfy the inequality $S_{\text{CFT}} > S > S_{\text{bh}}$ which proves that the holographic bound is not universal.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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